

```

dataSmall = {1, 1.5, 3, 5};
data = RandomReal[{0, 10}, 40];

KNormal[x_] := PDF[NormalDistribution[0, 1], x]
KUniform[x_] := PDF[UniformDistribution[{-0.5, 0.5}], x]

```

$$p[x_] := \frac{1}{\text{Length}[\text{dataSmall}] * h} \sum_{i=1}^{\text{Length}[\text{dataSmall}]} \text{KNormal}\left[\frac{\text{dataSmall}[[i]] - x}{h}\right]$$

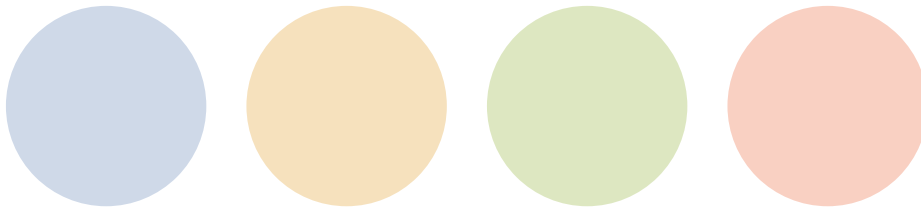
```
p[x] // Simplify
```

$$\frac{e^{-\frac{(-5+x)^2}{2h^2}} + e^{-\frac{(-3+x)^2}{2h^2}} + e^{-\frac{0.5(-1.5+x)^2}{h^2}} + e^{-\frac{(-1+x)^2}{2h^2}}}{4h\sqrt{2\pi}}$$

```

circles =
Graphics@
Table[{EdgeForm[], Opacity[0.3],
ColorData[97, "ColorList"][[i]], Disk[{2.4 * i, 0}],
{i, 1, 4}}]
dataLegend = SwatchLegend[{White}, {"s(x)"}],
LegendMarkers -> circles, LegendMarkerSize -> {10 * 4, 10}];

```



```

lines =
Graphics@
Table[{Opacity[0.5], ColorData[97, "ColorList"][[i]],
Line[{{i + 0.2, 0}, {i + 1, 0}]}, {i, 1, 4}}]
kernelLegend = SwatchLegend[{White}, {"K(t)"}],
LegendMarkers -> lines, LegendMarkerSize -> {16 * 4, 16}];

```

Calculation of  $h$  based on the average distance of the points.

```
DistanceMatrix[dataSmall] // MatrixForm
```

$$\begin{pmatrix} 0. & 0.5 & 2. & 4. \\ 0.5 & 0. & 1.5 & 3.5 \\ 2. & 1.5 & 0. & 2. \\ 4. & 3.5 & 2. & 0. \end{pmatrix}$$

```
selection = UpperTriangularize[ConstantArray[True, {4, 4}], 1];
selection // MatrixForm
```

$$\begin{pmatrix} 0 & \text{True} & \text{True} & \text{True} \\ 0 & 0 & \text{True} & \text{True} \\ 0 & 0 & 0 & \text{True} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
distances = Pick[DistanceMatrix[dataSmall], selection] // Flatten
{0.5, 2., 4., 1.5, 3.5, 2.}
```

```
0.5 * Mean[distances]
```

```
1.125
```

```
plot[data_, h_, K_, showHist_?BooleanQ, showKernels_?BooleanQ] :=
Block[{n = Length[data], colorPoints, p},
```

$$p[x_] := \frac{1}{\text{Length}[data] * h} \sum_{i=1}^{\text{Length}[data]} K\left[\frac{\text{data}[[i]] - x}{h}\right];$$

```
colorPoints = { ColorData[97, "ColorList"] showKernels;
{ Gray } True
```

```
colorPoints = Directive[#, Opacity[0.3]] & /@ colorPoints;
```

```
Show[
  ListPlot[Evaluate@Table[{{data[[i]], 0}}, {i, 1, n}],
    PlotStyle → colorPoints,
    PlotRange → {{-3, 10}, {-0.05, 0.55}},
    If[showKernels,
      PlotLegends → dataLegend /. {"s(x)" → "X"},
      PlotLegends → {"X"}
    ]
  ],

  If[showKernels,
    Plot[Evaluate@Table[0.1 * K[ $\frac{\text{data}[[i]] - x}{h}$ ], {i, 1, n}],
      {x, -3, 10},
      PlotRange → All,
      PlotStyle → Opacity[0.5],
      PlotLegends → kernelLegend
    ],
    {}
  ],

  If[showHist,
    Histogram[data,
      {Min[data], Max[data] + h, h},
      "PDF",
      ChartBaseStyle → Directive[Opacity[0.2],
        Specularity[White, 30]],
      ChartLegends → {"Histogram"}
    ],
    {}
  ],

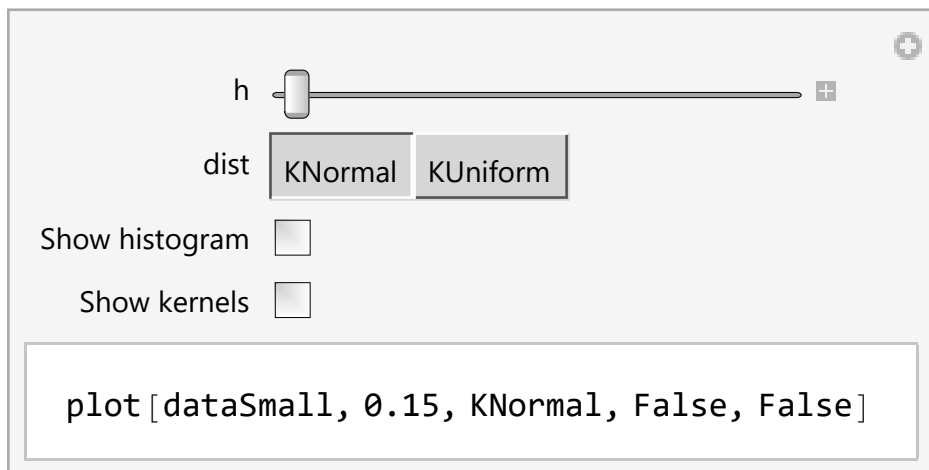
  Plot[p[x], {x, -1, 11}, PlotRange → All,
    PlotLegends → {" $\hat{p}(x)$ "},

    AxesLabel → {"x", " $\hat{p}(x)$ "},
    BaseStyle → {FontSize → 12},
    PlotLabel → "h = " <> ToString[h],
    ImageSize → Large,
    PlotRange → {{-1, 11}, {-0.05, 0.55}}
  ]
]
```

```
]
]
```

Manipulate[

```
plot[dataSmall, h, dist, showHist, showKernels]
, {h, 0.15, 2.5}, {dist, {KNormal, KUniform}},
{{showHist, False, "Show histogram"}, {True, False}},
{{showKernels, False, "Show kernels"}, {True, False}}]
```



The screenshot shows a Mathematica Manipulate interface. It features a slider for the parameter 'h' with a '+' icon on the right. Below the slider is a button menu for 'dist' with two options: 'KNormal' (selected) and 'KUniform'. There are two checkboxes: 'Show histogram' and 'Show kernels', both of which are currently unchecked. At the bottom, a text box displays the current command: `plot[dataSmall, 0.15, KNormal, False, False]`.

```

(*maxWidth=
ImageDimensions [
  Rasterize [plot [dataSmall,1,KNormal,True,True]] [[1]]

Do [
plots=
  Flatten [Table [plot [dataSmall,h,KNormal,showHist,
    showKernels], {h,0.2,2.5,0.05}],1];
plotsPadded= (
  ImagePad [
    Rasterize [#],
    {{0,maxWidth-ImageDimensions [Rasterize [#]] [[1]]},
    {0,0}},
    White
  ]
) &/@plots;

Export [
  FileNameJoin [ {
    NotebookDirectory [],
    "frames/showHist=" <> ToString [ { 0 showHist==False } <>
    { 1 showHist==True } <>
    "showKernels=" <> ToString [ { 0 showKernels==False } <>
    { 1 showKernels==True } <>
    "h=00.png"
  } ],
  plotsPadded,
  "VideoFrames",
  Antialiasing->True
];

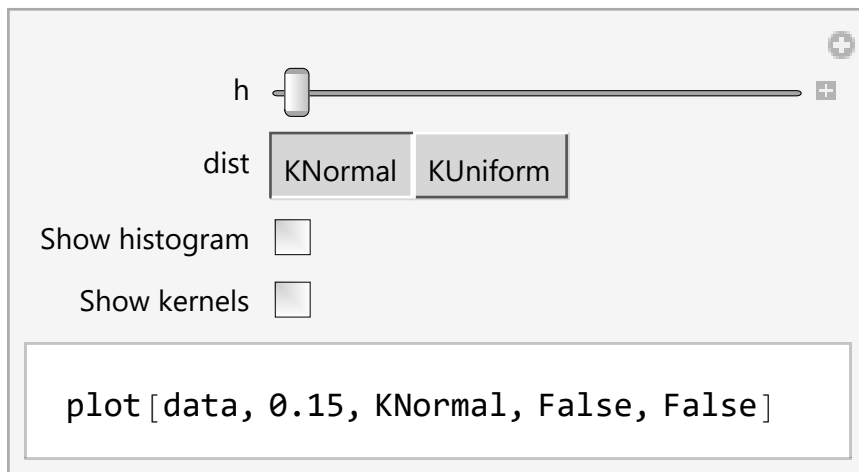
, {showHist, {True, False}}, {showKernels, {True, False}}
];*)

```

```

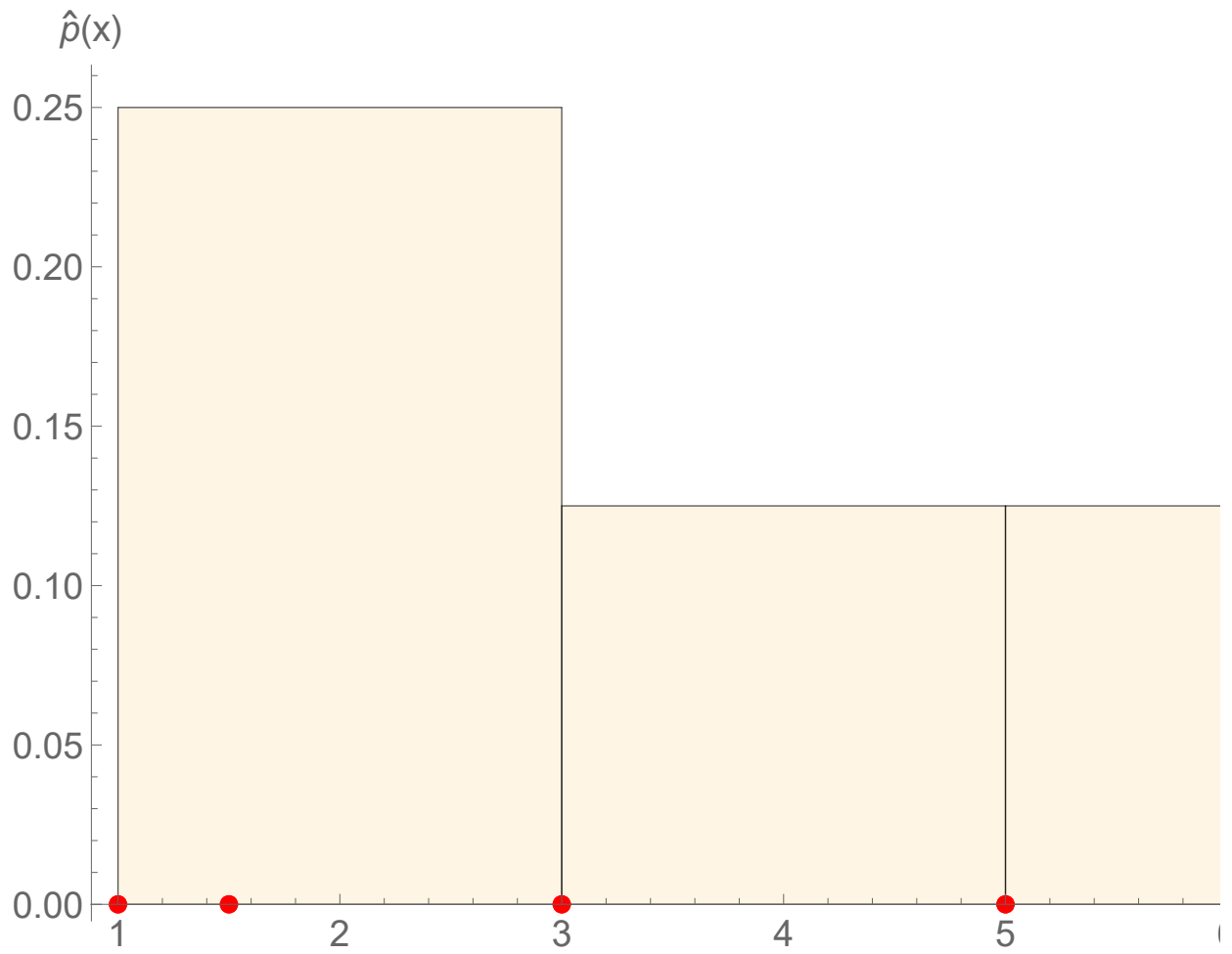
Manipulate [
  plot [data, h, dist, showHist, showKernels]
, {h, 0.15, 2.5}, {dist, {KNormal, KUniform}},
{{showHist, False, "Show histogram"}, {True, False}},
{{showKernels, False, "Show kernels"}, {True, False}}]

```



```
Show[
Histogram[dataSmall, {Min[dataSmall], Max[dataSmall] + 2, 2},
"PDF", ChartBaseStyle →
Directive[Opacity[0.2], Specularity[White, 30]],
ChartLegends → {"Histogram"},
LabelStyle → Directive[FontSize → 14]],
ListPlot[
Evaluate@Table[{{dataSmall[[i]], 0}},
{i, 1, Length[dataSmall]}],
PlotStyle → Directive[Red, PointSize[Large]],
PlotLegends → {"X"}, LabelStyle → Directive[FontSize → 14]],

ImageSize → Large,
AxesLabel → {"x", " $\hat{p}(x)$ "},
BaseStyle → {FontSize → 14}
]
```



$$\begin{aligned}
 \text{pUniform}[x\_] &:= \frac{1}{\text{Length}[\text{dataSmall}] * 2} \\
 &\sum_{i=1}^{\text{Length}[\text{dataSmall}]} \text{KUniform}\left[\frac{\text{dataSmall}[[i]] - x}{2}\right];
 \end{aligned}$$

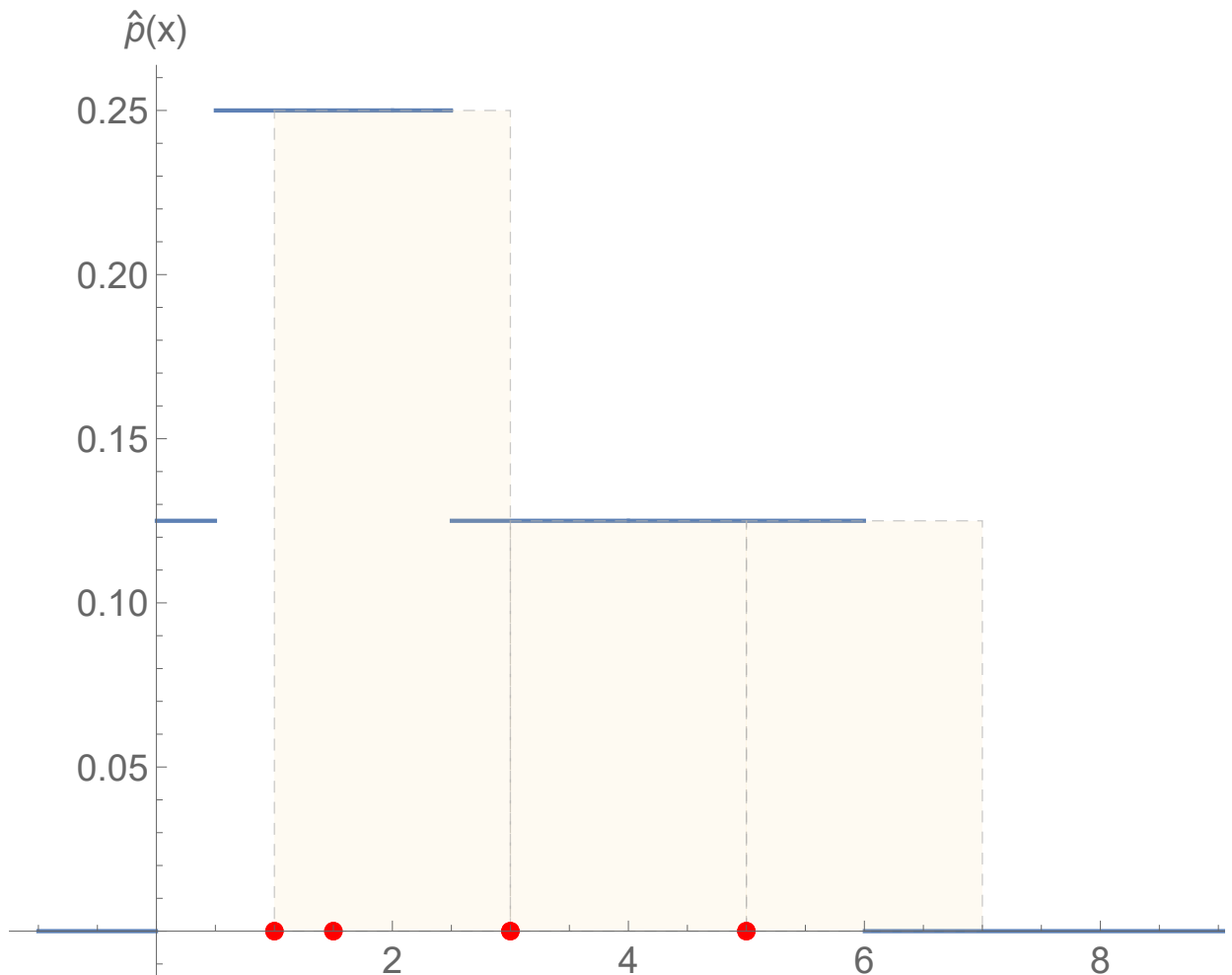
```

Show[
  Plot[pUniform[x], {x, -1, 11}, PlotRange → All,
    PlotLegends → {" $\hat{p}(x)$ "},
    LabelStyle → Directive[FontSize → 14]],
  Histogram[dataSmall,
    {Min[dataSmall], Max[dataSmall] + 2, 2},
    "PDF",
    ChartBaseStyle → {EdgeForm[{GrayLevel[0.7], Dashed}],
      Directive[Opacity[0.1], Specularity[White, 30]]},
    ChartLegends → {"Histogram"},
    LabelStyle → Directive[FontSize → 14]
  ],
  ListPlot[
    Evaluate@Table[{{dataSmall[[i]], 0}},
      {i, 1, Length[dataSmall]}],
    PlotStyle → Directive[Red, PointSize[Large]],
    PlotLegends → {"X"}, LabelStyle → Directive[FontSize → 14]],

  ImageSize → Large,
  AxesLabel → {"x", " $\hat{p}(x)$ "},
  BaseStyle → {FontSize → 14}
]

```





```
s[x_] = Plus @@ Table[DiracDelta[x - xi], {xi, dataSmall}]
```

```
DiracDelta[-5 + x] + DiracDelta[-3 + x] +  
DiracDelta[-1.5 + x] + DiracDelta[-1 + x]
```

```
p[x_] := 
$$\frac{1}{\text{Length}[\text{dataSmall}] * 1}$$
  

$$\sum_{i=1}^{\text{Length}[\text{dataSmall}]} \text{KNormal}\left[\frac{\text{dataSmall}[[i]] - \mathbf{x}}{1}\right];$$

```

```
pS[x_] = 
$$\frac{1}{\text{Length}[\text{dataSmall}]} * \text{Convolve}\left[\mathbf{s}[\mathbf{y}], \text{KNormal}\left[\frac{\mathbf{y}}{h}\right], \mathbf{y}, \mathbf{x}\right]$$

```

$$\frac{e^{-0.5(1.5-1.\mathbf{x})^2/h^2} + e^{-(-5+\mathbf{x})^2/2h^2} + e^{-(-3+\mathbf{x})^2/2h^2} + e^{-(-1+\mathbf{x})^2/2h^2}}{4\sqrt{2\pi}}$$

```

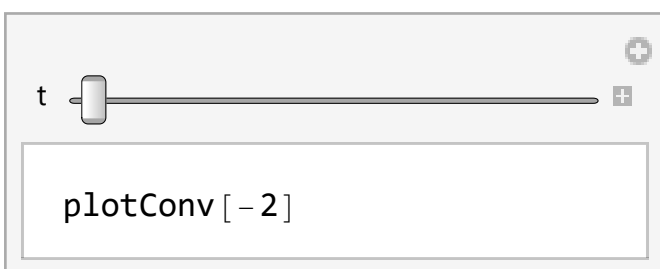
plotConv[t_] := Show[
  ListPlot[
    Evaluate@Table[{{dataSmall[[i]], 0.2 * KNormal[0]}},
      {i, 1, Length[dataSmall]}],
    PlotStyle → (Directive[#, Opacity[0.3]] &) /@
      ColorData[97, "ColorList"],
    Filling → Axis,
    PlotLegends → dataLegend
  ],
  Plot[Evaluate[Table[
    {
      0.2 * KNormal[ $\frac{x - \text{dataSmall}[[i]]}{1}$ ]  $x \leq t$ ,
      Undefined  $\text{True}$ 
    }
    {i, 1, Length[dataSmall]}]
  ]], {x, -3, 10},
    PlotRange → All,
    PlotStyle → Opacity[0.5],
    PlotLegends → kernelLegend
  ],
  Plot[
    {
      pS[x] /. {h → 1}  $x \leq t$ ,
      Undefined  $\text{True}$ 
    }, {x, -3, 10},
    PlotLegends → {"(K*s) (t)"}
  ],
  AxesLabel → {"x", " $\hat{p}(x)$ "},
  PlotLabel → "h = 1",
  AxesOrigin → Automatic,
  ImageSize → Large,
  BaseStyle → {FontSize → 12},
  PlotRange → {{-2, 10}, {0, 0.3}}
]

```

```

Manipulate[
  plotConv[t]
, {t, -2, 10}]

```



```
(*Export[
  FileNameJoin[{
    NotebookDirectory[],
    "frames/t=00.png"
  }],
  Table[plotConv[t],{t,-2,10,0.25}],
  "VideoFrames",
  Antialiasing→True
];*)
```